# 8.2 - Analyzing Arithmetic Sequences and Series



## Warmup

| Arithmetic | Geometric |
|---|---|
| $S_n = \frac{n}{2}(a_1 + a_n)$ | $S_n = \frac{a_1(1 - r^n)}{1 - r}$ |

Find the sum.

1. $\sum_{t=0}^{4} (13 + 7t)$ 135

2. $\sum_{i=1}^{5} (1 + 7i)$ 110

3. $\sum_{p=3}^{7} (2p - 1)$ 45

4. $\sum_{j=0}^{6} (24 - 9j)$ $-21$

5. $\sum_{b=2}^{6} (2b + 1)$ 45

6. $\sum_{y=5}^{11} (3y - 5)$ 133

7. $\sum_{i=1}^{7} 2i$ 56

8. $\sum_{r=3}^{6} (r + 2)$ 26

# 8.4 - Finding Sums of Infinite Geometric Series



The sum $S_n$ of the first n terms of an infinite series is called a **partial sum**. The partial sums of an infinite geometric series may approach a limiting value.

Consider the infinite geometric series: $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots$

The partial sums are

$$S_1=\frac{1}{2}=0.5$$

$$S_2=\frac{1}{2}+\frac{1}{4}=0.75$$

$$S_3=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\approx 0.88$$

$$S_4=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}\approx 0.94$$

$$S_5=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}\approx 0.97$$

$$S_6=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}\approx 0.98$$

# 8.4 - Finding Sums of Infinite Geometric Series



Using the previous slide, consider the sum of a finite geometric series:

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right) = \frac{1}{2}\left(\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}\right) = 1-\left(\frac{1}{2}\right)^n$$

As n increases, $\left(\frac{1}{2}\right)^n$ approaches 0, so $S_n$ approaches 1.

# 8.4 - Finding Sums of Infinite Geometric Series



In general, if $0 < |r| < 1$ then as n increases

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right) \approx a_1\left(\frac{1 - 0}{1 - r}\right) = \frac{a_1}{1 - r}.$$

**The Sum of an Infinite Geometric Series**

The sum of an infinite geometric series with first term $a_1$ and common ratio $r$ is given by

$$S = \frac{a_1}{1 - r}$$

provided $|r| < 1$. If $|r| \geq 1$, then the series has no sum.

# 8.4 - Finding Sums of Infinite Geometric Series



What if $r > 1$ or $r < -1$?

i.e. let $r = 2$ and $r = -2$

$r = 2$

$5, 10, 20, 40, 80, \ldots \rightarrow a_\infty$

$r = -2$

$5, -10, 20, -40, 80, \ldots \rightarrow a_\infty$

NO SUM

$100, 50, 25, 12.5, 6.25, \ldots \rightarrow a_\infty$

What is r?

$r = \frac{1}{2}$

What is the sum of this series?

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a_1(1 - r^\infty)}{1 - r} = \frac{a_1(1 - 0)}{1 - r} = \frac{a_1}{1 - r}$$

$$S_\infty = \frac{100}{1 - \frac{1}{2}} = 200$$

# 8.4 - Finding Sums of Infinite Geometric Series



**The Sum of an Infinite Geometric Series**

The sum of an infinite geometric series with first term $a_1$ and common ratio $r$ is given by

$$S = \frac{a_1}{1 - r}$$

provided $|r| < 1$. If $|r| \geq 1$, then the series has no sum.

Practice - Find the sum of each infinite geometric series.

a. $\sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^{n-1}$

$\frac{2}{3}$

b. $3 + \frac{3}{4} + \frac{3}{16} + \frac{3}{64} + \ldots$

$4$

c. $\frac{4}{3} - \frac{2}{3} + \frac{1}{3} - \frac{1}{6} + \ldots$

$\frac{8}{9}$

# 8.4 - Finding Sums of Infinite Geometric Series



Creating Fractions

$$0.124 = \frac{124}{1000} = \frac{31}{250}$$

$$3.255 = \frac{3255}{1000} = \frac{651}{200}$$

Repeating Decimals

$$0.131313... = 0.\overline{13}$$

$$x = 0.131313...$$

$$100x = 13.131313...$$

$$100x - x = 13$$

$$99x = 13$$

$$x = \frac{13}{99}$$

$$2.\overline{1230} =$$

$$x = 2.\overline{1230} \qquad 10000x = 21230.\overline{1230}$$

$$10000x - x = 21228$$

$$9999x = 21228$$

$$x = \frac{21228}{9999} = \frac{7076}{3333}$$

# 8.4 - Finding Sums of Infinite Geometric Series



Repeating Decimals - Special Number

$$x = 0.\overline{9}$$

$$10x = 9.\overline{9}$$

$$10x - x = 9$$

$$9x = 9$$

**x = 1**

# 8.4 - Finding Sums of Infinite Geometric Series



## Repeating Decimals

$$0.131313... = 0.\overline{13}$$

$$x = \frac{13}{99}$$

$$2.\overline{1230} =$$

$$x = \frac{21228}{9999} = \frac{7076}{3333}$$

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## Practice

1) $0.\overline{324}$

$$\frac{12}{37}$$

2) $3.\overline{0141}$

$$\frac{10046}{3333}$$

3) $12.\overline{9876}$

$$\frac{43288}{3333}$$